Quantum Simulation using Optical Lattices

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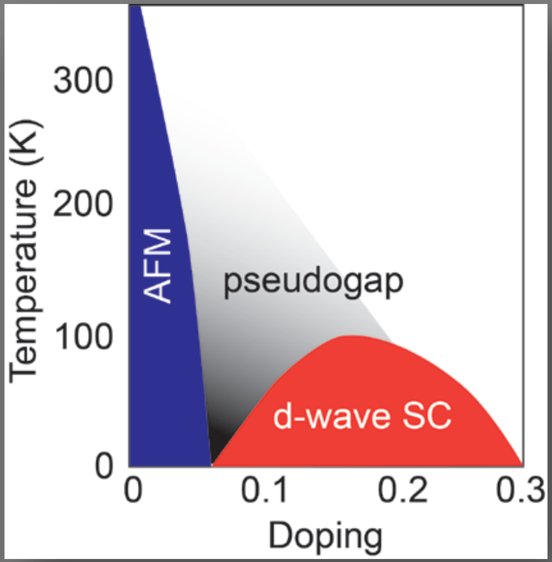
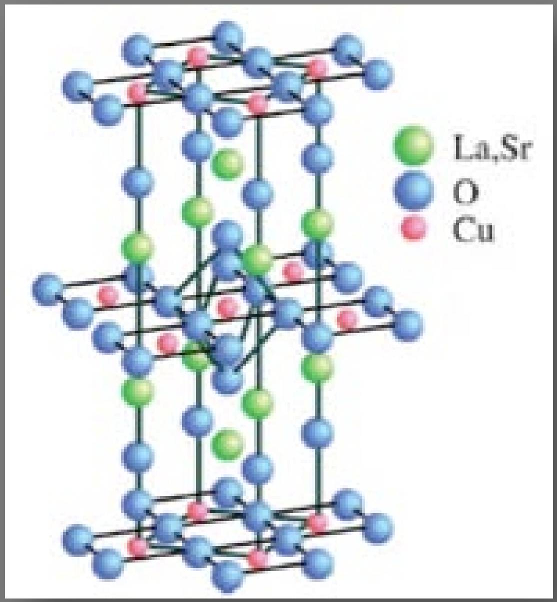
Agenda:

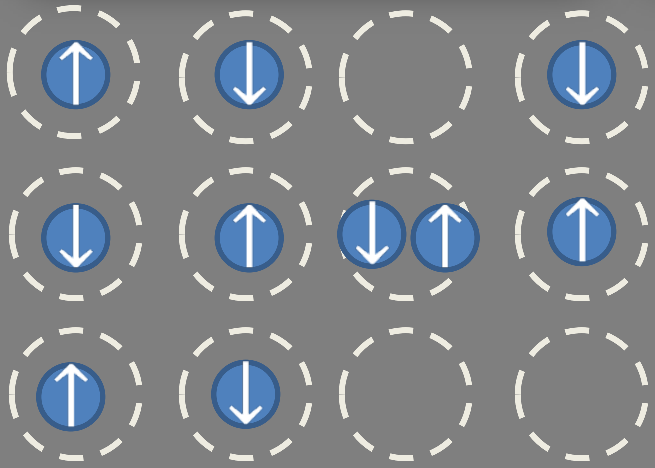
- Basic Physics of Atoms in Optical Lattices
- Thermometry and Cooling; Spin-Dependent Lattices
- Disordered Gases

The Premise: an example

High temperature superconductivity / Hubbard model







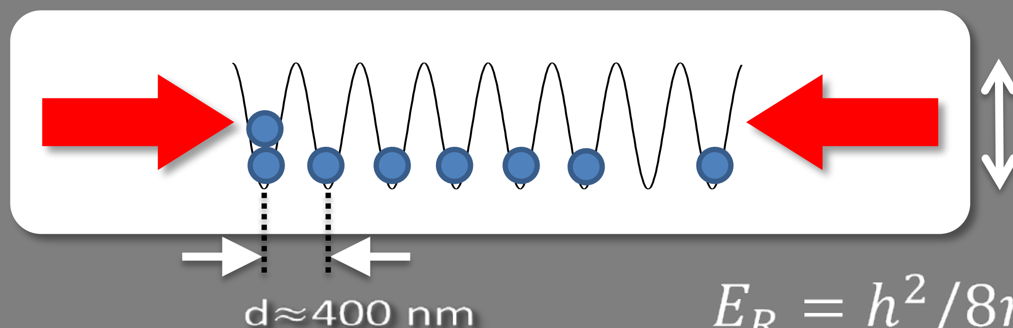
$$H = -t \sum_{\langle ij \rangle, \sigma} c^{\dagger}_{i,\sigma} c_{j,\sigma} + U \sum_{i} n_{i,\uparrow} n_{i,\downarrow}$$

tunneling — interactions

- Can Hubbard model produce d-wave SC?
- Nature of pseudo-gap phase?

Optical Lattice Quantum Simulation

Atoms confined by periodic potential arising from intensity and/or polarization gradients



$V_{lat} = sE_R$

$E_R = h^2/8md^2$

$\approx 170\, nK \cdot k_B$

"recoil energy"

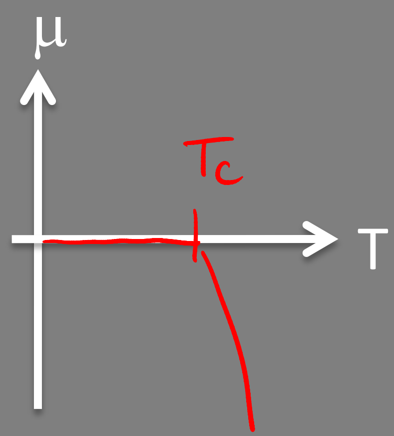
"Exactly" realizes the Hubbard model

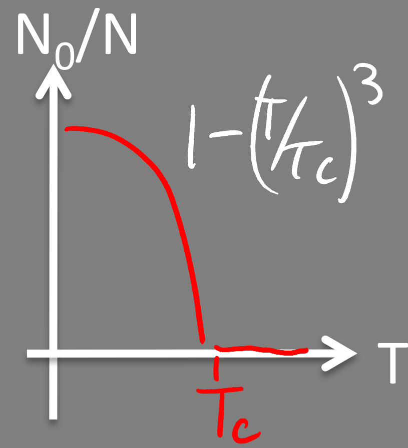
Today:

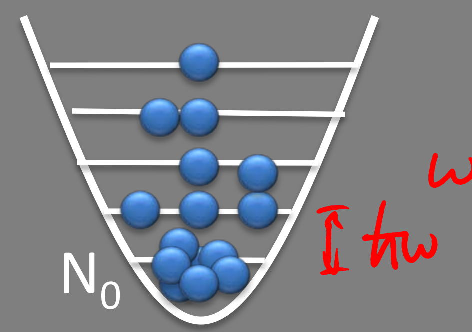
- A on little cooling / quantum degeneracy
- Optical lattice potentials / AC Stark Effect
- Interactions (collisions)

Quantum Degeneracy (for an ideal gas)

Bosons



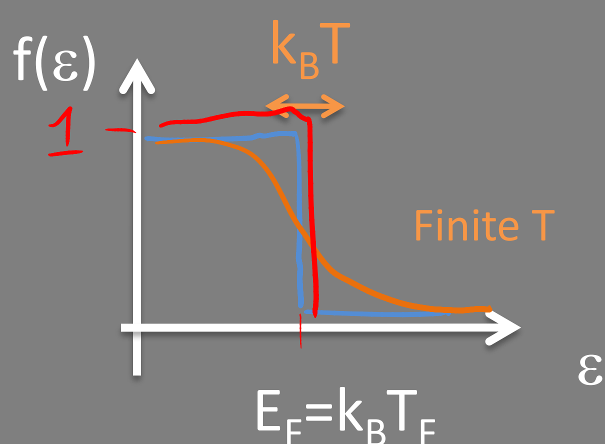




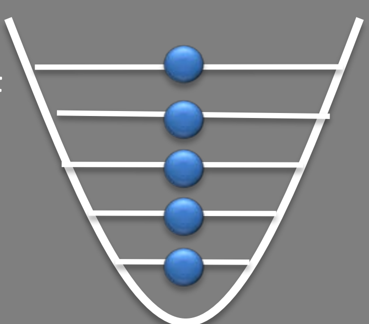
$$T_c = 0.94\hbar\omega N^{1/3} \qquad n_{pk}\Lambda_{dB}^3 = 2.6$$

Fermions

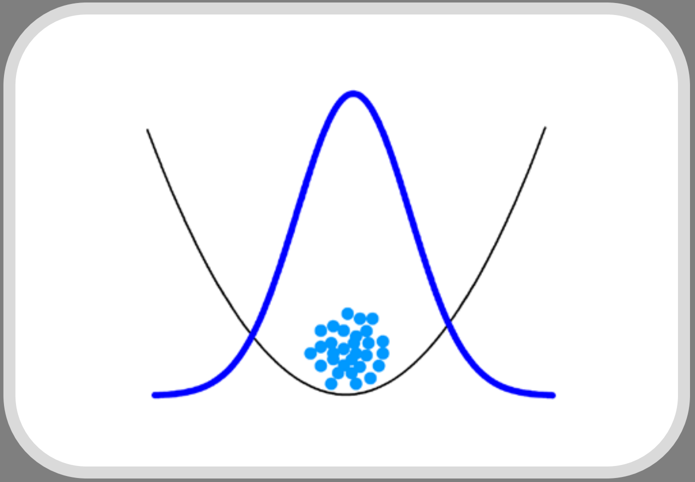
$$E_F = \hbar\omega(6N)^{\frac{1}{3}}$$



$E_F = k_B T_F$

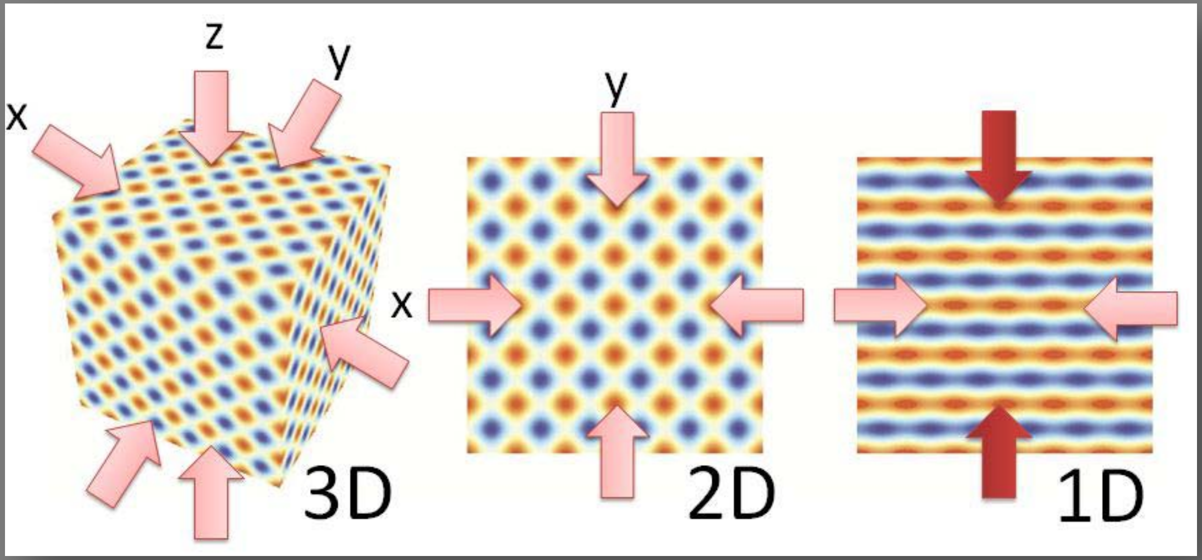


Lattice Potentials



Superimpose lattice potential on trapped, degenerate gas

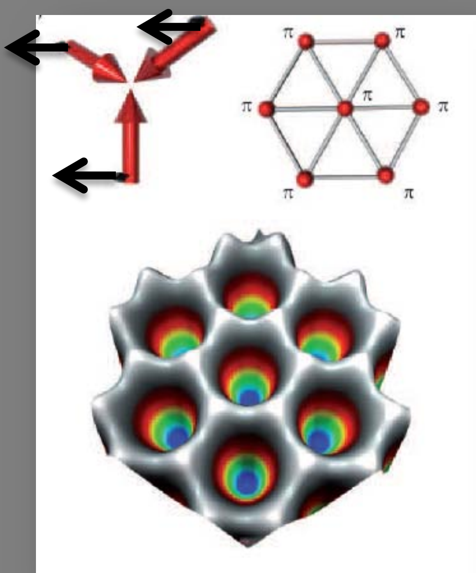
Simplest geometry: cubic, square, 1D



Other geometries

triangular

hexagonal



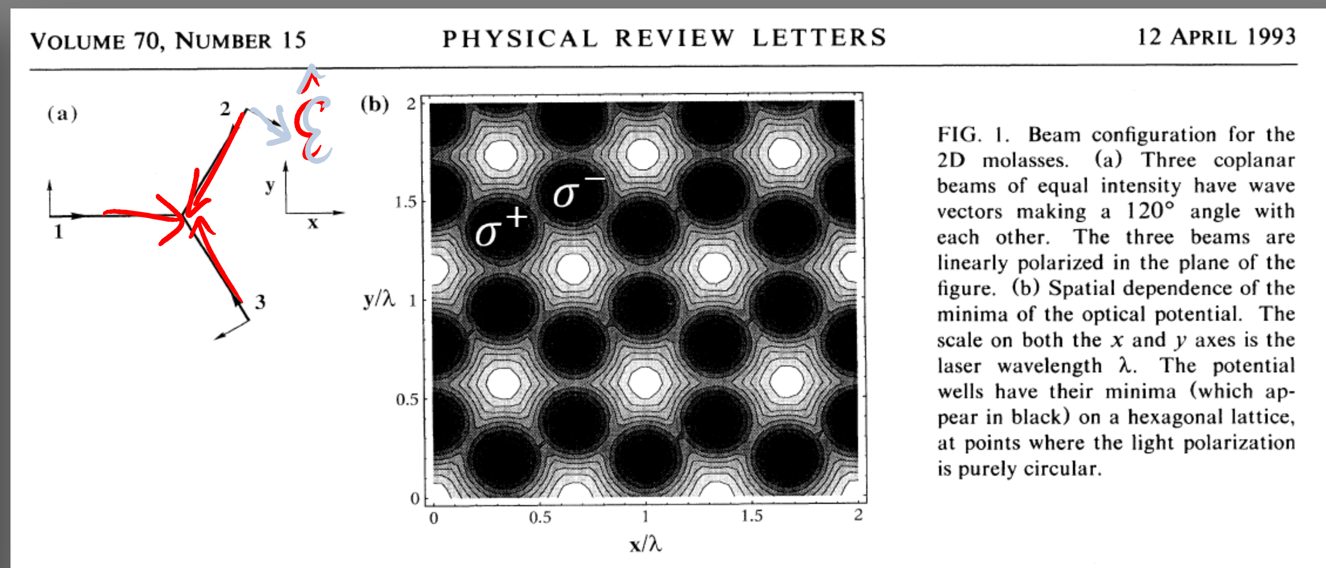

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FIG. 1. Beam configuration for the 2D molasses. (a) Three coplanar beams of equal intensity have wave vectors making a 120° angle with each other. The three beams are linearly polarized in the plane of the figure. (b) Spatial dependence of the minima of the optical potential. The scale on both the x and y axes is the laser wavelength λ. The potential wells have their minima (which appear in black) on a hexagonal lattice, at points where the light polarization is purely circular.

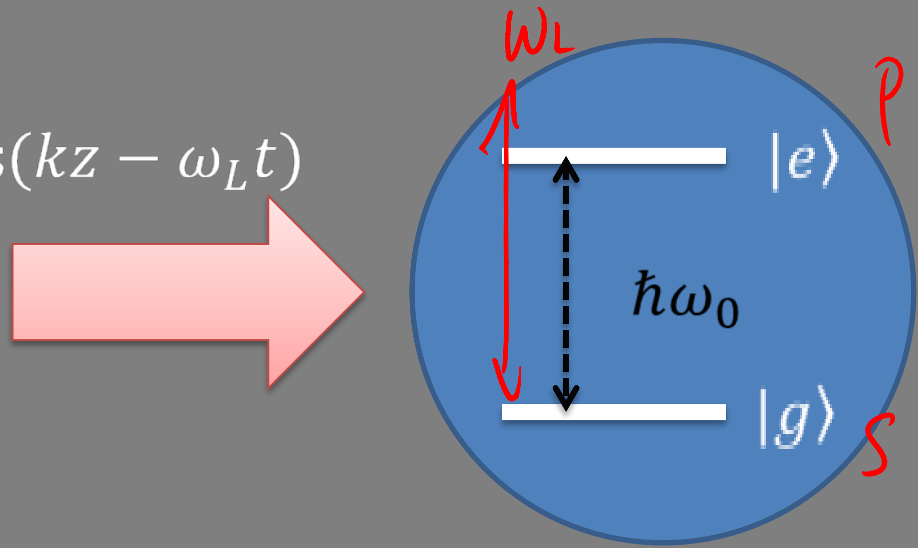
Sengstock

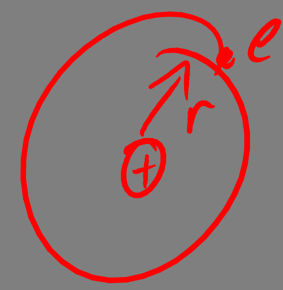
Spin-dependent: different F, m_F experience different potentials
(more tomorrow)

AC Stark Effect

$$\vec{E} = E_0 \hat{\epsilon} \cos(kz - \omega_L t)$$

polarization





$$H_I = e\vec{r} \cdot \vec{E}\cos(\omega_L t)$$

dipole matrix element

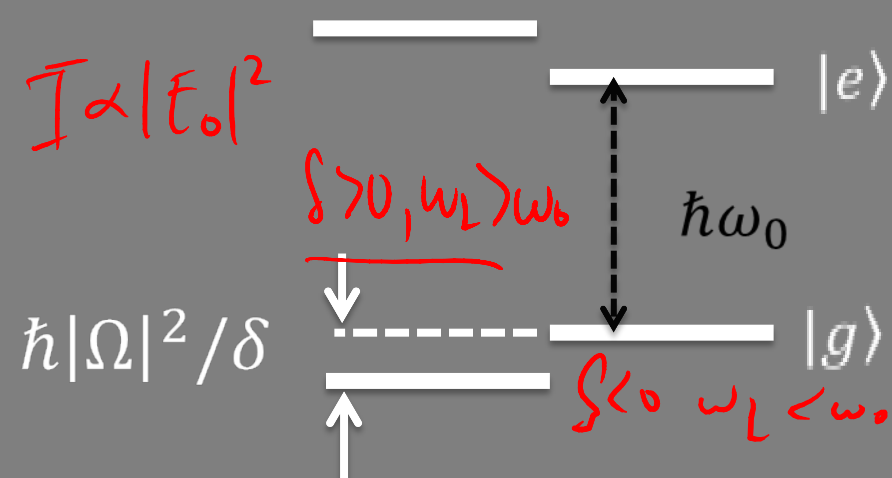
Treat atom as 2-level system
(electric dipole approximation)

Rabi frequency: $\Omega = \frac{eE_0}{2}\langle e|\vec{r}\cdot\hat{\epsilon}|g\rangle$

Rotating wave approximation,
$\Omega \ll \delta = \omega_L - \omega_0$

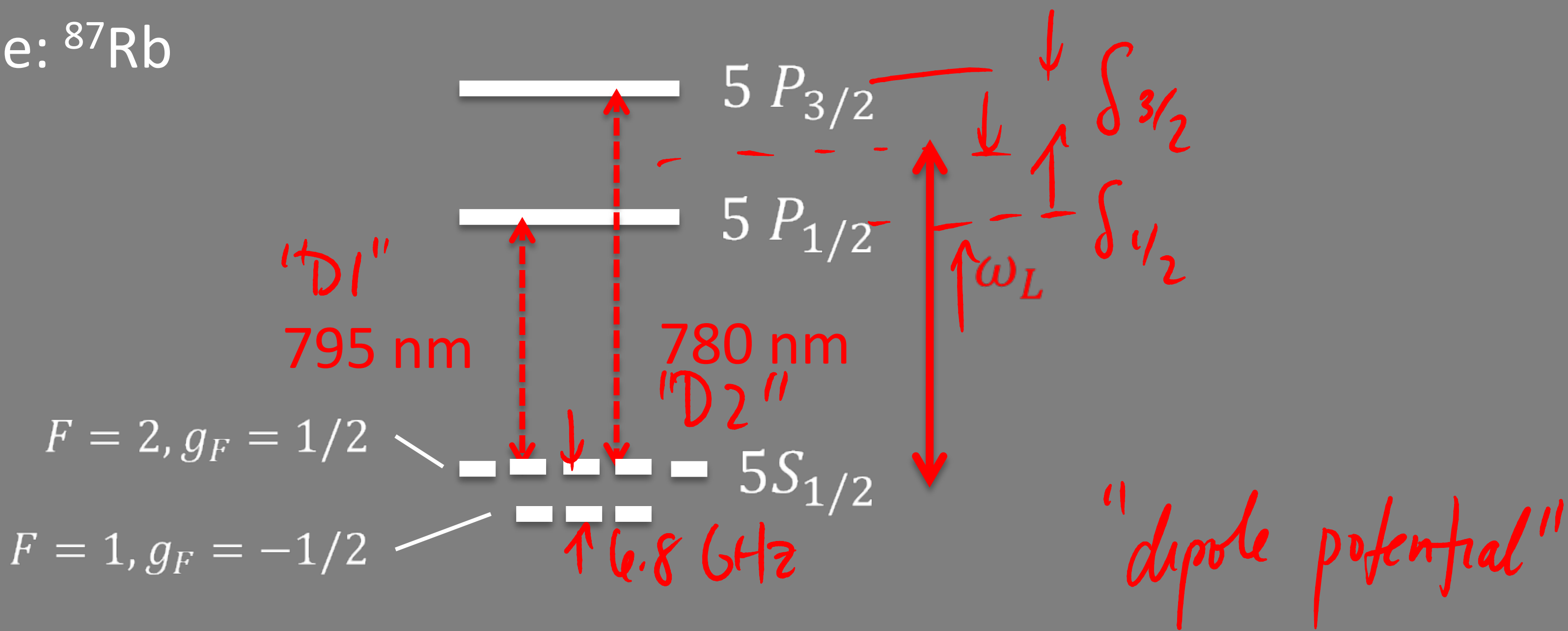
$I \propto |E_0|^2$

$\Delta E \propto I/\delta$



Real Atoms

Example: ^{87}Rb



$$U_{dip} = \frac{\pi c^2 \Gamma}{2\omega_0}\left(\frac{2 + P g_F m_F}{\delta_{3/2}} + \frac{1 - P g_F m_F}{\delta_{1/2}}\right) I(\vec{r})$$

$|\Omega|^2 \propto |E_0|^2$

$P = -1\ (\sigma^-), 0\ (\pi), 1\ (\sigma^+)$ Polarization ***in the atomic basis***

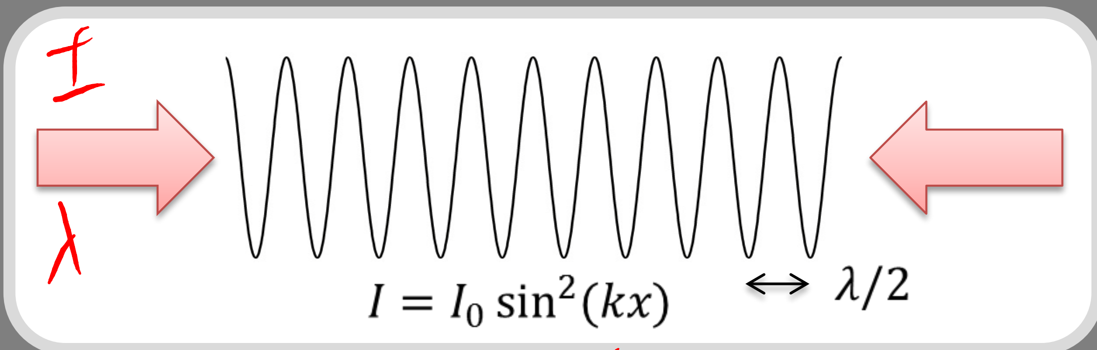
$$\Gamma = \frac{\omega_0^3}{3\pi\epsilon_0 \hbar c^3} |\langle e|e\vec{r}|g\rangle|^2 = 1/\tau \approx 2\pi \times 6\ MHz$$

Spin-Independent Lattice

π-polarized light (or linear in any basis), or large detunings

1064 nm

$$U_{dip} = \frac{3\pi c^2 \Gamma}{2\omega_0 \delta} I(\vec{r})$$



$U \propto \sin^2(kx)$

Red detuning:

$\delta < 0$, trapped in anti-nodes

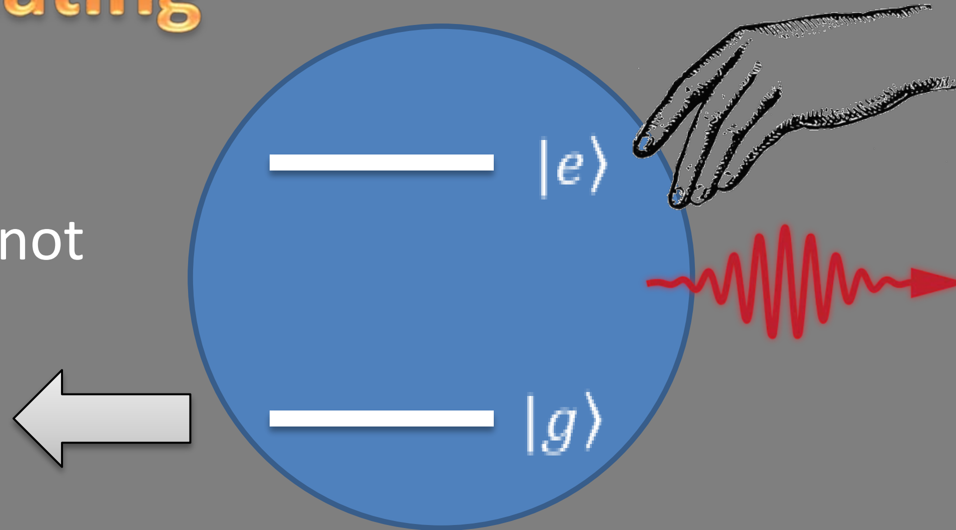
Blue detunings:

$\delta > 0$, trapped in nodes

Note corrections for large detunings
($\lambda = 532, 1064, 1550\ nm$)

$$U_{dip} = -\frac{3\pi c^2 \Gamma}{2\omega_0}\left(\frac{1}{\omega_0 - \omega_L} + \frac{1}{\omega_0 + \omega_L}\right) I(\vec{r})$$

Light-induced Heating



Electromagnetic vacuum is not our friend (sometimes)

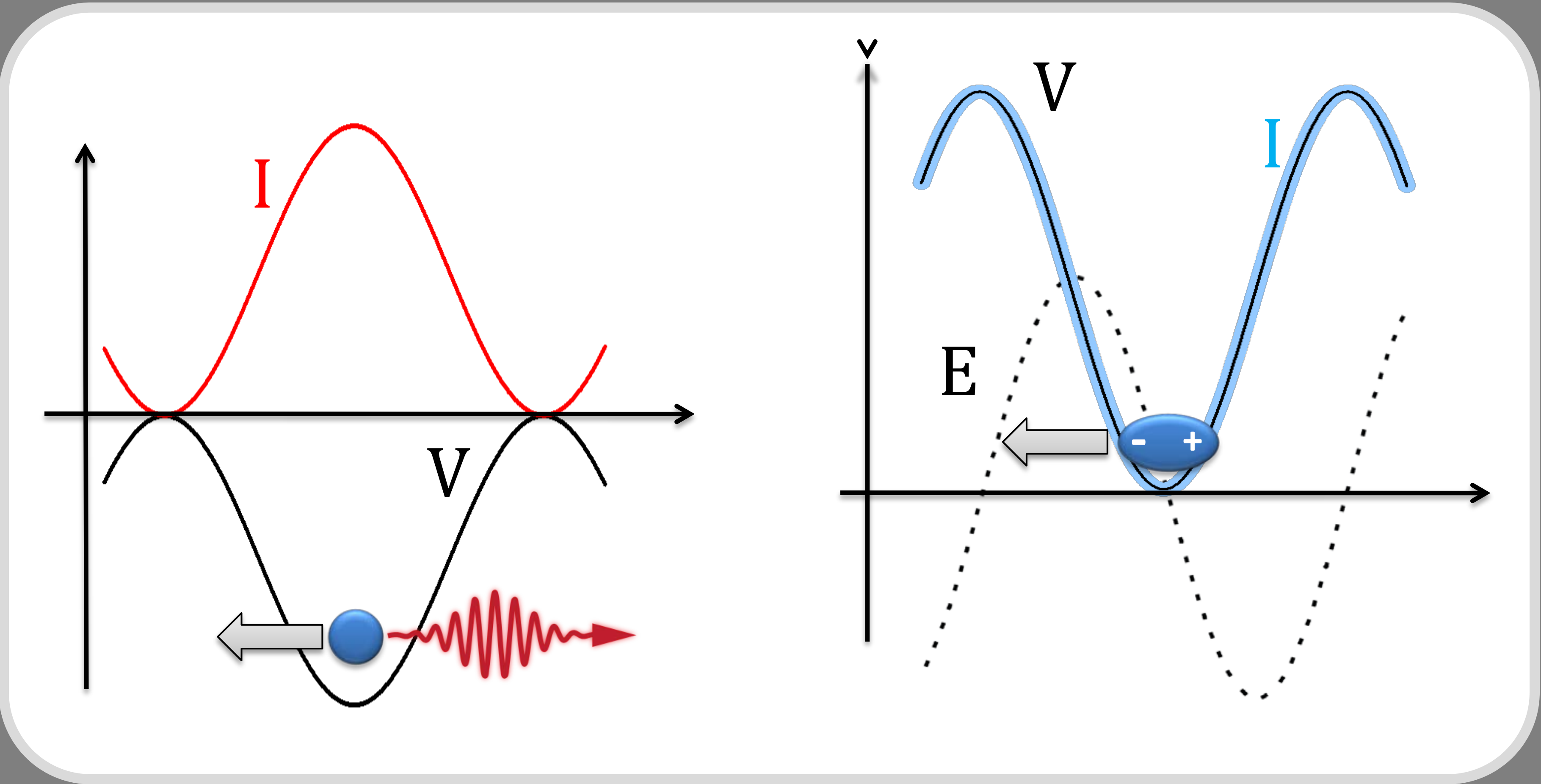
So, it's better to keep the atoms in the dark in a blue-detuned lattice, right?

Gordon and Ashkin: it doesn't matter

$$\dot{E} = \frac{E_R 6\pi c^2 \Gamma^2}{\hbar \omega_0^3 \delta^2} I_0$$

$$\dot{E} = \frac{E_R 6\pi c^2 \Gamma^2}{\hbar \omega_0^3} I_0 \left(\frac{\omega}{\omega_0}\right)^3 \left[\frac{1}{(\omega_0 - \omega_L)^2} + \frac{1}{(\omega_0 + \omega_L)^2}\right]$$

Light Induced Heating



Result the same for spin-dependent lattices, imbalanced beams, beams at an angle…

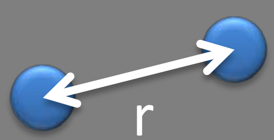
Ultra-cold collisions

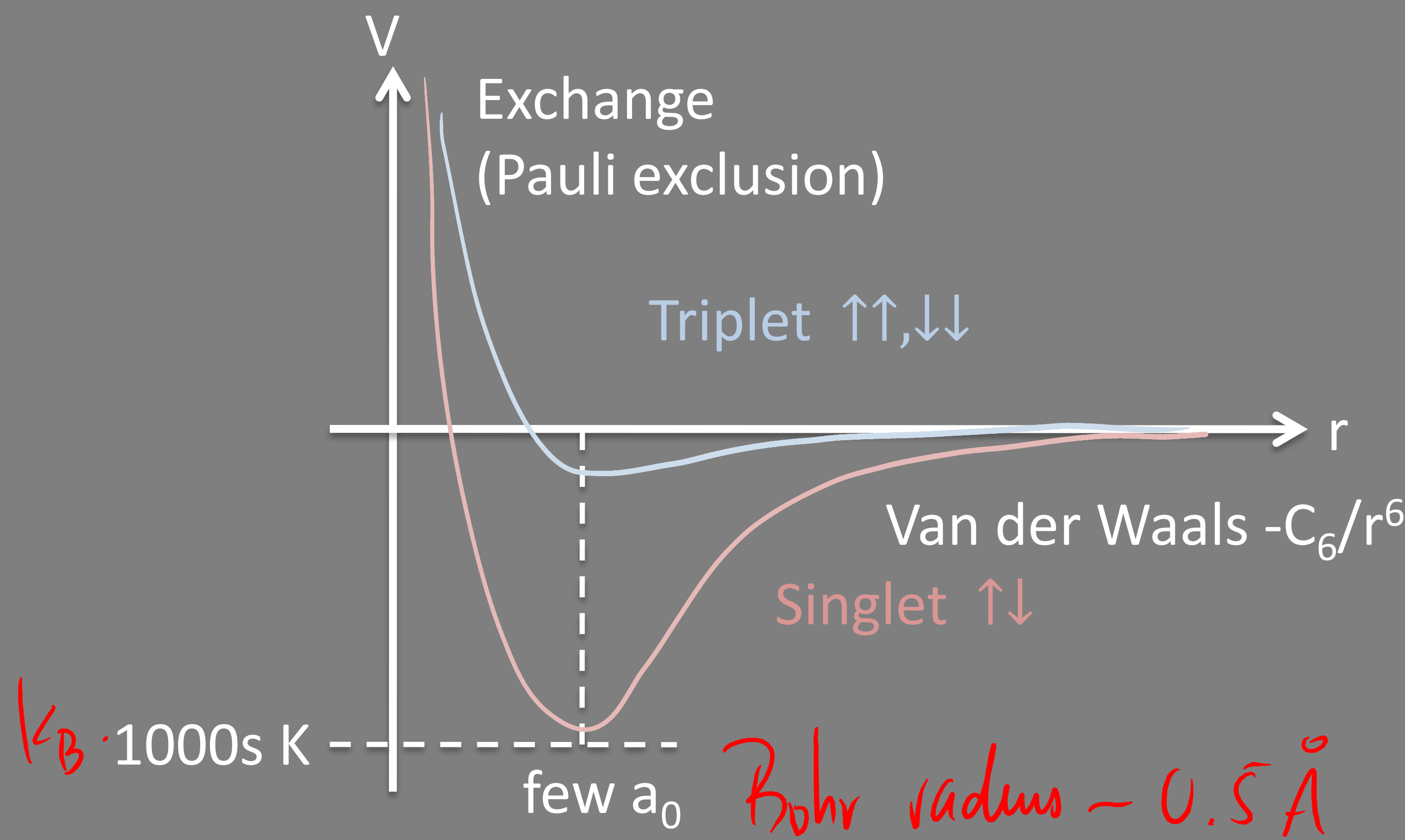
$$H = -t\sum_{\langle ij\rangle,\sigma} c^{\dagger}_{i,\sigma} c_{j,\sigma} + U\sum_{i} n_{i,\uparrow} n_{i,\downarrow}$$

Where does U come from?

a: scattering length

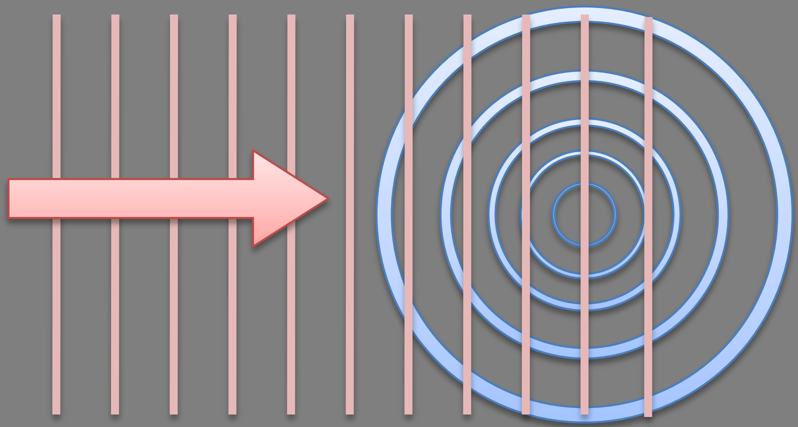
One number that characterizes binary, low-energy collisions!





Ultra-cold collisions

Very basic quantum scattering theory



Goal: find the scattered wave

Lippman – Schwinger

$$\Psi \propto e^{ikz} + f(k,\Omega)\frac{e^{ikr}}{r}$$

f: scattering amplitude

$$\Psi = \sum_l R_l(k,r) P_l(\cos\vartheta)$$

partial wave expansion

$$\left[\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} - \frac{l(l+1)}{r^2} - V(r) + k^2\right] R_l(k,r) = 0$$

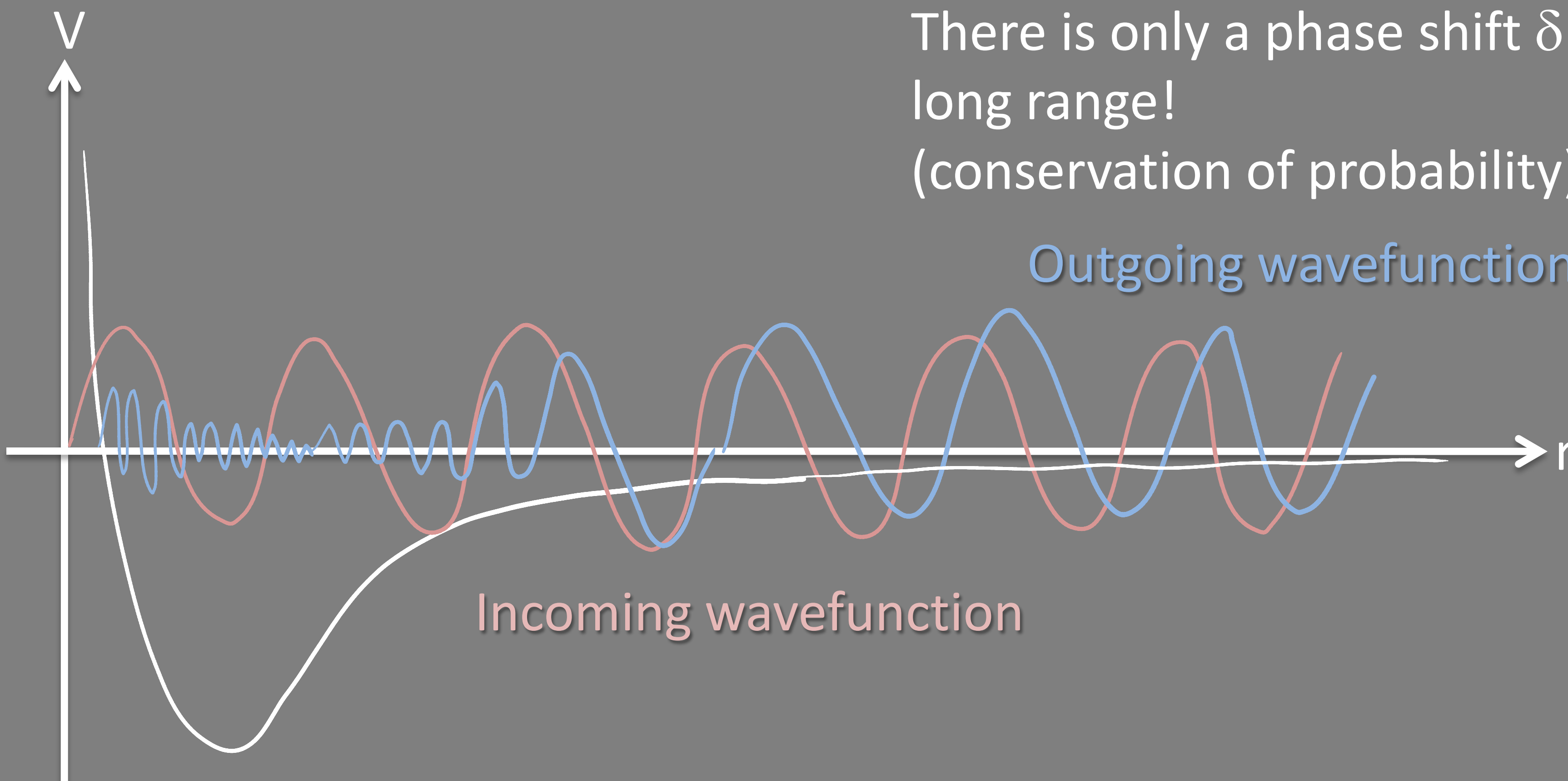
centrifugal barrier

Ultra-cold collisions

Very basic quantum scattering theory

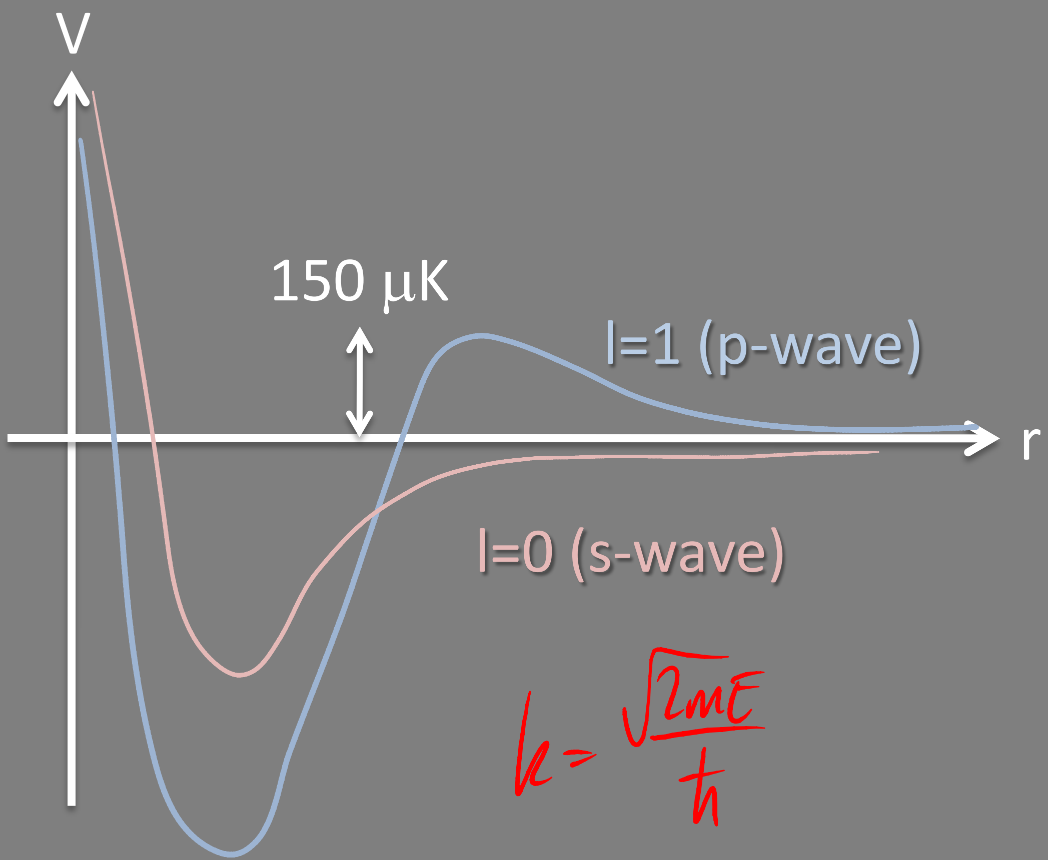
Asymptotic solution $$R_l \underset{r\to\infty}{\propto} \frac{1}{kr}\sin\left[kr - l\frac{\pi}{2} + \delta_l(k)\right]$$

There is only a phase shift δ at long range!
(conservation of probability)



Ultra-cold collisions

Very basic quantum scattering theory



$$k = \frac{\sqrt{2mE}}{\hbar}$$

At ultra-cold temperatures, we only have to consider *s*-wave (l=0) collisions

$$R_0 \underset{r\to\infty}{\propto} \sin\left(kr + \delta_0\right)/kr$$

$$R_0 \underset{r\to\infty}{\propto} \sin\left[k\left(r-a\right)\right]/kr$$

Trade phase shift for a length

$$a = -\lim_{k\to 0}\frac{\delta_0\left(k\right)}{k}$$

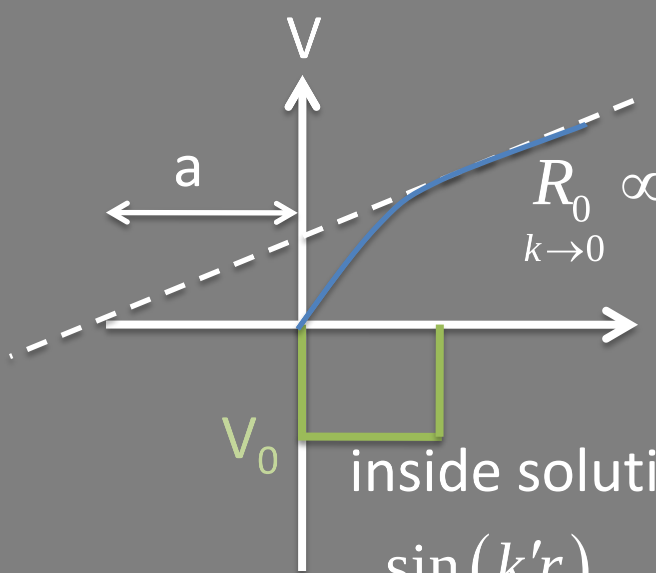
$$a = -\infty \to \infty$$

Ultra-cold collisions

Very basic quantum scattering theory

Examples: ^{133}Cs a=-3000 a_0 ^{40}K a=200 a_0 (at B=0)

How can a take on any value?

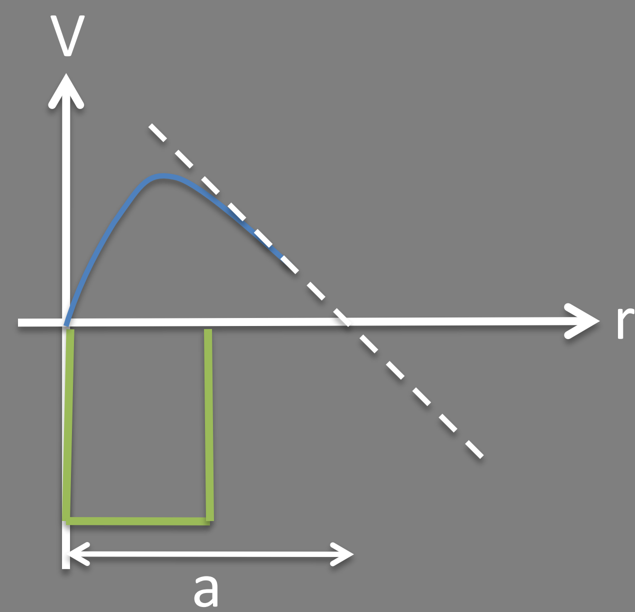


$$\underset{k\to 0}{R_0} \propto 1 - a/r$$

inside solution

$$\sin(k'r)$$

$$k' = \sqrt{2mV_0/\hbar}$$



a >0 or <0 related to last bound state energy

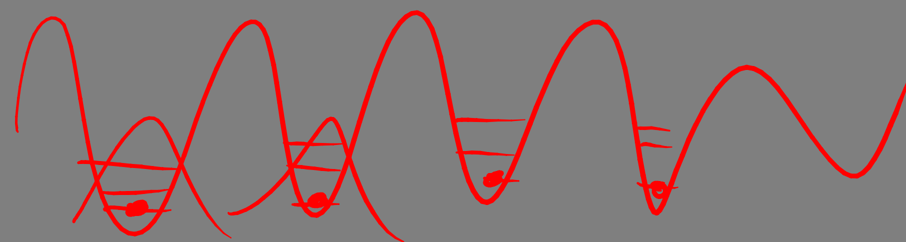
…Feshbach resonances on Friday

Contact interaction approximation

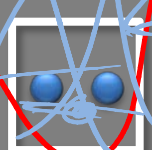
Interactions treated using a pseudo-potential (see Huang)

$$V(r) = \frac{4\pi\hbar^2}{m} a\delta^3(\vec{r})$$

- Long range behavior correct $R \propto 1 - a/r$
- Enforces boundary condition $\Psi(r = a) = 0$



Calculating U



$$E_{\text{int}} \propto \int d^3\vec{x}_1 \int d^3\vec{x}_2 \phi_0^*(\vec{x}_1)\phi_0^*(\vec{x}_2)\frac{4\pi\hbar^2 a}{m}\delta^3(\vec{x}_1 - \vec{x}_2)\phi_0(\vec{x}_1)\phi_0(\vec{x}_2)$$

$$U \propto \frac{4\pi\hbar^2 a}{m}\int d^3\vec{x}\left|\phi_0(\vec{x}_1)\right|^4$$

Ultra-cold collisions

Very basic quantum scattering theory

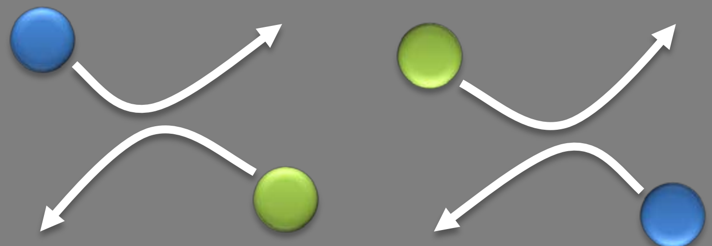
bosons vs fermions

$$\Psi = \Psi_{space} \left| \chi_{spin} \right\rangle \propto P_l(\cos\vartheta) \left| \chi_{spin} \right\rangle$$

Two-particle wavefunction must have proper exchange symmetry

$\Psi(x_1, x_2) \rightarrow \Psi(x_2, x_1)$ bosons

$\Psi(x_1, x_2) \rightarrow -\Psi(x_2, x_1)$ fermions



equivalent to 180° rotation

$P_l(\cos\vartheta) = P_l(-\cos\vartheta)$ l even

$P_l(\cos\vartheta) = -P_l(-\cos\vartheta)$ l odd

Ultra-cold collisions

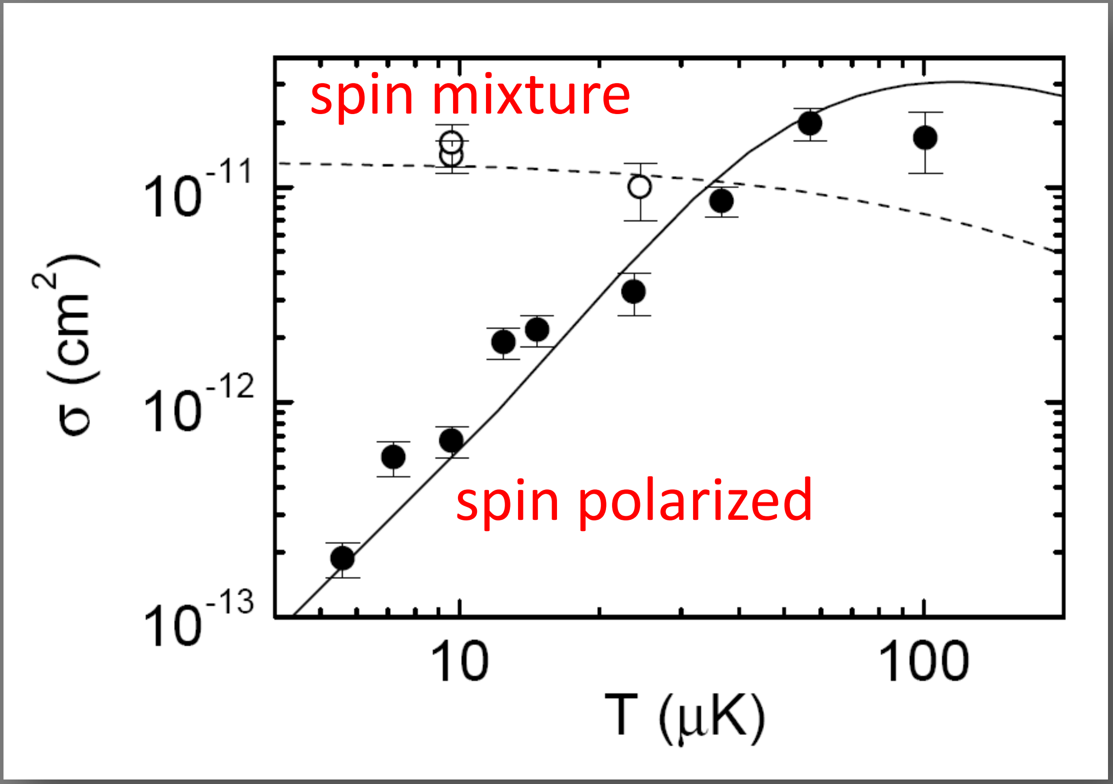
Very basic quantum scattering theory

At low temperatures, l=0: the spatial wavefunction must be symmetric

$$|\chi\rangle = \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

Fermions: require spinor to be anti-symmetric

Multiple spin states are required for fermionic atoms to interact!



References

- *Bose-Einstein Condensation in Dilute Gases*, Pethick & Smith
- *Atom Optics*, Meystre
- *Atomic Physics*, Budker, Kimball, DeMille
- *Laser Cooling and Trapping*, Metcalf
- *Bose-Einstein Condensation*, Pitaesvskii & Stringari
- *Atomic Physics*, Foot
- *Bose-Einstein Condensation in Atomic Gases*, Proc. of Fermi Summer School, Ed. Inguscio, Stringari, Wieman
- DeMarco graduate course notes, http://online.physics.uiuc.edu/courses/phys598map/spring08/
- MIT lecture notes, http://cua.mit.edu/8.421/
- JILA lecture notes, http://www.colorado.edu/physics/phys7550/phys7550_sp07/
- Dan Steck alkali data notes, http://steck.us/alkalidata/
- McKay and DeMarco, *New J. Phys.* **12** 055013 (2010)